# Composite Materials: Analysis and Design

## Homework no.1

### Problem 1

Prove the following relations:

I. $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla}.\vec{A}) - \nabla^2\vec{A}$

II. $(\vec{A} \times \vec{B}).(\vec{C} \times \vec{D}) = \begin{vmatrix} \vec{A}.\vec{C} & \vec{A}.\vec{D} \\ \vec{B}.\vec{C} & \vec{B}.\vec{D} \end{vmatrix}$

III. $(\vec{A} \times \vec{B}).(\vec{B} \times \vec{C}) \times (\vec{C} \times \vec{A}) = [\vec{A}.(\vec{B} \times \vec{C})]^2$

IV. $(\vec{A} \times \vec{B}).(\vec{C} \times \vec{D}) = [\vec{A}.(\vec{C} \times \vec{D})]B - [\vec{B}.(\vec{C} \times \vec{D})A]$

### Problem 2

An anisotropic elastic solid is subjected to some load that gives a strain state $\varepsilon_{ij}$ in the $x_1x_2x_3$ coordinate system. In a different (rotated) coordinate system $x_{1'}x_{2'}x_{3'}$ the strain state is transformed to $\varepsilon_{m'n'}$.

(a) Do you expect the strain energy density function $U_0$ to be a function of strain invariants only?

(b) Do you expect the same or different expressions of $U_0$ when it is expressed in terms of $\varepsilon_{ij}$ or $\varepsilon_{m'n'}$?

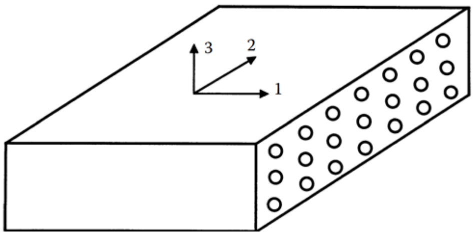
(c) Do you expect the same or different numerical values of $U_0$ when you compute it from its expression in terms of $\varepsilon_{ij}$ and from its expression in terms of $\varepsilon_{m'n'}$?

(d) Justify your answers.

Answer parts (a), (b) and (c) if the material is isotropic.

**Problem 3**

Show the reduction of orthotropic material stress-strain Equation to those of a transversely isotropic material stress-strain Equation.




**Problem 4**

(a) A thin triangular plate is fixed along the boundary OA and is subjected to a uniformly distributed horizontal load $p_0$ per unit area along the boundary AB as shown in the figure. Give all boundary conditions in terms of displacement or stress components in $x_1x_2$ coordinate system.

(b) If $p_0$ acts normal to the boundary AB what will be the stress boundary conditions along line AB?